Dynamic analysis of the effect of immigration on the demographic background of the pay-as-you-go pension system

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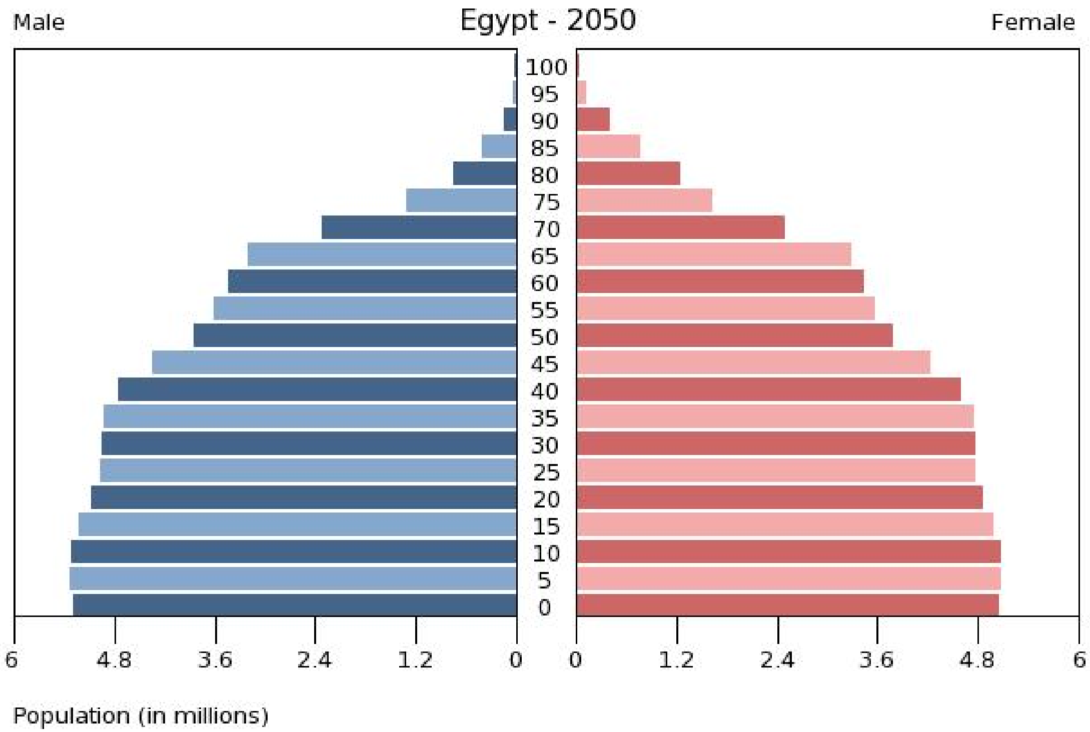
Problem Setting and motivation

Demographic equilibrium as a *key variable* to ensure the sustanaibility of a pay-as-you-go pension system.

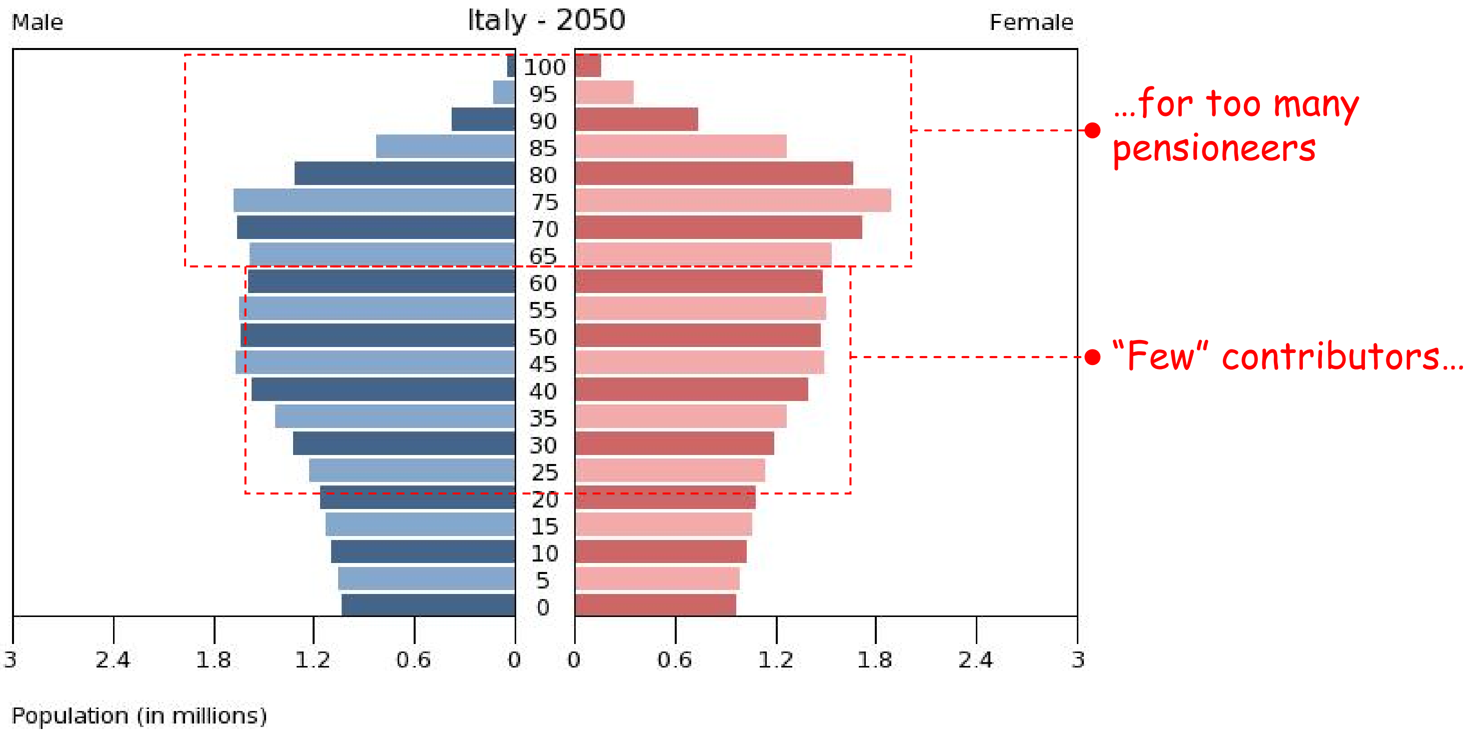
"Ideal" shape



Problem Setting and motivation

The problem is that population pyramids of mature economies often display "critical" features

"Bad" shape



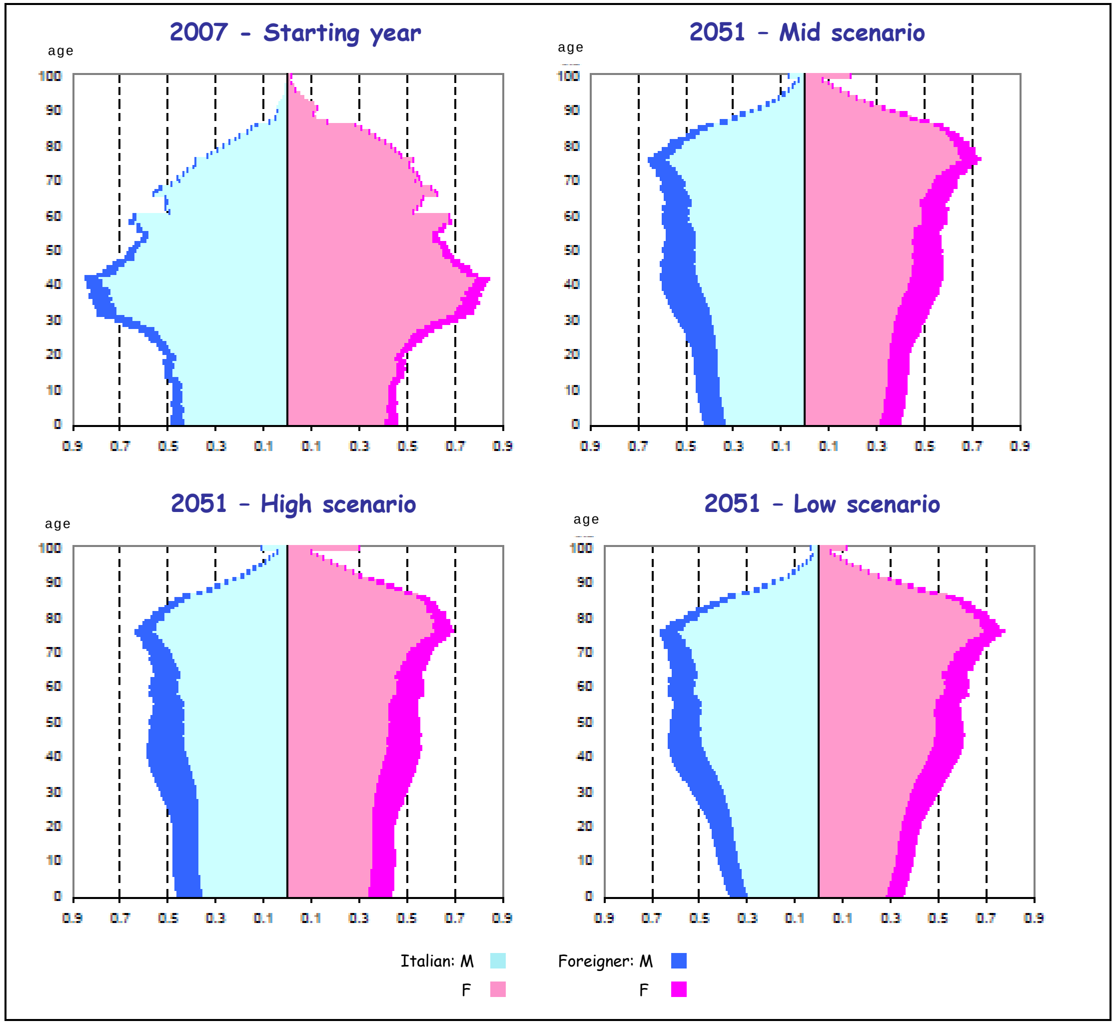
Problem Setting and motivation

With respect to previous estimates, ISTAT forecasts that the Italian population will grow more than 6 million people as a consequence of immigration.

In 2051 immigrants will represent 16,1%-18,4% of the whole population.

The basic idea:

Immigrants as a resource for stabilizing the population distribution in order to achieve the sustainability of the pay-as-you-go pension system.



Problem Setting and motivation

Pros

Rejuvenating the age structure of the population owing to two main reasons:

- Immigrants are generally young (*immediate effect*)
- Immigrants generally display ferility rates higher than the Italian one (*postponed effect*)

ISTAT Fertility rates (mid scenario)

	2007	2051
Italian	1,36	1,39
Immigrants	2,35	1,86

Convergence to the reproductive behaviour of the Italian females

Remark

The analysis of the demography for the pension system is the very first step towards a more general solution also including the economic component.

The basic population model

Consider a population without sex structure where:

- $N \in$ N is the upper bound of the age of an individual;
- $x_i(t)$ is the number of individuals of age belonging to $[i, i+1[$ (for $i = 0, 1, \dots, N-1$) at time t;
- $\alpha_i \geq 0$ average per capita birth rate in the i-th age group;
- $0 < \omega_i < 1$ is the survival rate from age group i to $i+1$;

The population vector results $x(t) = \left[x_0(t),\ x_1(t),\ \dots, x_{N-1}(t)\right]^T$

and the system matrix is

$$L = \begin{bmatrix} \alpha_0 & \alpha_1 & \dots & \alpha_{N-2} & \alpha_{N-1} \\ \omega_0 & 0 & \dots & 0 & 0 \\ 0 & \omega_1 & & 0 & 0 \\ . & . & \dots & . & . \\ 0 & 0 & & \omega_{N-2} & 0 \end{bmatrix}$$

The population dynamics (so called Leslie model) is $x(t+1) = L\ x(t)$

The modified population model

The Leslie model ➡ Insects

Our modified Leslie model ➡ Human population

Let

- N be the maximal age for each sex;
- $x^F(t)$ be the female population vector;
- $x^M(t)$ be the male population vector;
- x(t) be the whole population vector;
- f_i be the per mil fertility;
- q_i^F be the female mortality rate;
- $\omega_i^F = 1 - q_i^F$ be the female survival rate;
- $\varphi = \dfrac{x_0^F(T)}{x_0^F(T)+x_0^M(T)} = \dfrac{x_0^F(T)}{x_0(T)}$ be the sex ratio (at birth)

The female per capita birth rate is $\alpha_i^F = \dfrac{f_i}{1000}\varphi$ (i=15,…,50)

The modified population model

The modified Leslie female matrix becomes

$$L^F = \begin{bmatrix} 0 & 0 & \dots & \alpha^F{}_{15} & \dots & \alpha^F{}_{50} & 0 & \dots & 0 & 0 \\ \omega_0^F & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & \omega_1^F & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ . & . & \dots & . & \dots & . & . & \dots & . & . \\ 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & \omega_{N-2}^F & 0 \end{bmatrix}$$

The dynamics of females reads as $x^F(t+1) = L^F x^F(t)$

The modified Leslie male matrix becomes

$$L^M = \begin{bmatrix} 0 & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \omega_0^M & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & \omega_1^M & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ . & . & \dots & . & \dots & . & . & \dots & . & . \\ 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & \omega_{N-2}^M & 0 \end{bmatrix}$$

The dynamics of males reads as $x^M(t+1) = L^M x^M(t) + \dfrac{(1-\varphi)}{\varphi} e_1 \circ L^F x^F(t)$